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A systematic approach on analyzing the relationship between straightness & angular errors and guideway surface in precise linear stage☆

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ABSTRACT

In this paper, a systematic approach on how to calculate the straightness and angular errors based on measuring guideway surface and fitting curve is introduced. Straightness and angular errors play an important role in precise system, which can undermine the system accuracy, especially in multi-axis motion structure. Conventional method adopts a periodic function to represent the guideway surface. However, for majority environments, the guideway surface curve is random depending on different machining processes. Thus, it is necessary to develop a precise method to obtain the guideway surface curve for calculating the straightness & angular errors. Instead of adopting trigonometric function to represent the guideway surface, this paper measures the guideway first. By analyzing the characteristics of machining process for guide rail, the proper characteristic functions are selected in curve fitting based on the measurement results, and an accurate analytical expression of guideway surface is obtained. Therefore, the SaA error values can be calculated with corresponding formulas based on the expression. Compared with previous method, the new approach is more accurate in curve fitting and error calculation, which can be applied in other similar environment through the same procedure. Furthermore, by analyzing the measured results of guideway surface, the new approach procedure can be regarded as a bridge between the pattern of measured guideway surface and corresponding manufacture process, which is seldom discussed in other research works. This new approach is also comprehensive and systematic in error analysis in precise linear stages, which is beneficial to derive the distribution of straightness and angular errors for engineers before installation in design part. A case study of a precise linear stage by following the procedure in the new approach is developed, and the comparison between calculation and measured results proves the validation of the new approach.

1. Introduction

The kinematic error in motion system includes positioning error, straightness error and angular error (pitch, yaw and roll). Majority scholars concentrated on the positioning error because of applications [1–3]. However, the straightness and angular errors (SaA) also play an essential role in stage. In most engineering regions, the influence of straightness and angular errors are important in a complicated system, such as numerical center and multi-axis motion platform. In these multi-axis systems, the lower body can give impact on the pose of subsequent stages, so the SaA errors not only affect positioning accuracy at other directions, but also undermine the whole system accuracy, especially if SaA errors are sensitive errors. In consequence, an error analysis approach about SaA error calculation is required to solve these problems.

Different from the positioning error which is affected by motor, lead screw etc., the SaA errors are determined by the quality of guideway surface and assembly error etc. Xue [4] gave a brief introduction about error identification in hydrostatic guideways. The error averaging effect is an essential part in accuracy improvement. However, the guideway surface accuracy is not in consideration in this paper. Hwang [5] adopted a three-probe system to measure the parallelism and straightness of a pair of rails for ultra-precision guideway. Although this paper was aiming at error measurement, it provided a structure analysis and parallelism algorithm for straightness error identification. Previous studies focused on SaA error in different aspects, and some scholars mentioned SaA errors in geometric error modeling [6–8]. Florussen et al. [9] pointed out the relationship between joint kinematic straightness and angular errors, which could be used for error model robustness improvement of error model. Brecher et al. [10] presented a

☆ Research Area: Error transferring and precision analysis for multi-axis system.

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Nomenclature

SaA errors Straightness and Angular errors
PLS Precise Linear Stage
X, Y, Z, U, V, W Axis in PLS
Δx, △y, △z, △u, △v, △w Kinematic error
y_i, (P) Coordinate of the point to be calculated (midpoint of carriage)
$y_{i\text{-}1}$, y_{i+1}, (A, B) Two contact points between carriage and guideway
K Length of carriage
f Fitting curve function of guideway surface
$\triangle x_c, \triangle y_c, \triangle z_c, \triangle u_c, \triangle v_c, \triangle w_c$ Kinematic error by calculation
$\triangle x_m, \triangle y_m, \triangle z_m, \triangle u_m, \triangle v_m, \triangle w_m$ Kinematic error by measurement
g_1, g_2 Two trends of guideway surface curve

mathematical method which can be used for SaA errors deduction in high precision multi-axis machine design. The evaluation of joint geometric errors has also been attracted scholars' attention. Liu [11] proved that in a translational motion element, the perpendicularity error is a combination including straightness and angular error. For multi-axis machine tools, the compensation of translational and rotational errors is different. The conclusion indicated that the modeling of perpendicularity is complicated. Ekinci et al. [12] carried out an experiment in researching relationship between SaA errors. The author firstly gave a systematic error classification about different error definitions, and a simulation between guideway and errors was introduced. However, when focusing on the ratio between length of carriage and curve of surface of guideway, this paper regarded a trigonometric function as the surface curve of guideway, which was not suitable for representing the pattern of guideway curve in real situation. It is easy for SaA error calculation under this assumption, but not accurate and effective. He also established an error model in matrix form based on guideways' geometric error and the motion errors theoretically and experimentally [13]. Another group of researches concentrated on load effect and deformation. Zha [14] took a gantry type open hydrostatic guideway as an example to research the straightness error modeling and compensation. Through measuring the straightness error in different points on the beam, a static analysis model is established, which is beneficial for error compensation. Majda [15] presented some problems in geometric errors. He adopted Finite Element Method (FEM) in joint geometric error in linear guideway modeling, and gave a deformation comparison between FEM results and producers characteristics. But his work did not discuss validation of the method in small size stage. Pawelko [16] raised a methodological basis of modeling roller guides with preload, which can be used for installation guidance.

Majority studies paid attention to long scale guideways which are widely used in machine tools, including geometric error and deformation effect. However, the influence by the surface of guideway is lack of systematic research. First, few scholars established a link between guideway surface and guideway manufacturing process. Due to different processes in machining guideway, the curve of guideway surface is irregular. Second, considering the surface of guideway is random, how to obtain a precise curve for guideway surface is significant.

In this paper, a systematic approach on the relationship between guideway surface and SaA errors in precise linear stage (PLS) is proposed. Through measuring guideway surface, a fitting curve can be obtained instead of trigonometric function in conventional method. This curve not only represents guideway surface precisely, but also provides informative guidance related to guideway manufacture process. By carrying out the analytical procedure and simulation in a mathematical way through the new approach, the SaA errors can be calculated with corresponding formulas. A series of experiments about a PLS and a pair of cross roller guide are introduced to validate this approach. This approach not only benefits for error compensation in complicated system, but also helpful in accuracy improvement. Furthermore, the distribution of SaA can also be known in advance by this method, which can provide information for engineers in design part.

2. Geometric error definitions

Generally, geometric error includes kinematic error and location/assembly error [2]. The former part is related to motion state, and the latter part is resulted from installation. For example, when a translational motion system moves along Y axis, there are 6 directional kinematic errors, including 3 translational axes (X, Y, Z) are named positioning error Δy and straightness errors Δx &Δz (at other two axes), and 3 rotational axes (U, V, W) named angular errors (pitch Δu, yaw Δw and roll Δv), respectively, as is shown in Fig. 1. The physical meaning of each error is related to moving direction in Table 1.

On one hand, the positioning error is caused by power accuracy (such as motor force, material character), assembly error, and machining error of each part, etc. On the other hand, the SaA errors are affected by the machining error of guide rail, the assembly error of each part. According to Fig. 1, the pitch Δu rotates in Z-Y plate, which links to Z directional straightness error Δz. Both two errors are affected by Z directional deviation. Thus, in this paper, the following content will discuss how the Z directional deviation give impact on Z directional straightness error Δz and the pitch error Δu. and the horizontal straightness error Δx and angular error yaw Δw could be derived with the same analyzing procedure. The roll error is difficult in error identification in papers, which is affected by parallelism of guide rail as well. The kinematic error classifications are listed in Table 2.

Besides the kinematic error, the location error is from non-orthogonality in two DOF installations. Theoretically, it contains six directional errors in one DOF, 3 translational axes and 3 rotational axes. However, most of the location errors can neglect, and only rotational error needs to be considered.

3. SaA errors analysis in PLS

In recent research area, PLS is widely adopted in various high-accuracy-requirement regions, such as optoelectronic packaging system, magnet levitation motion system, femtosecond laser manufacturing process etc. Compared with machine tools, the PLS is smaller, more accurate and agile, which is used for controlling the volumetric orientation precisely. This paper takes the guide rail in PLS as a research object, and a typical structure of one PLS is illustrated in Fig. 2. It contains motor, coupler, screw, guideway and carriage etc. It may be a little difference depends on various needs and situations.

In this kind of PLS, the kinematic error is determined by the motor power accuracy, the machining error of coupler & lead screw, and the assembly error among couple, lead screw, guideway and carriage. Similarly, as SaA errors are affected by guideway and assembly errors, an analysis about demonstrating the relationship between SaA errors and carriage-guideway are required. For majority PLS, the carriage is considered to be supported by two rigid bearing, which is moving along

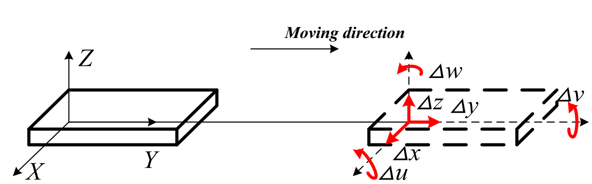


Fig. 1. Six directional errors in a translational axis.

Table 1
Physical meanings of 6 kinematic errors in a translational axis (moving along Y-axis).

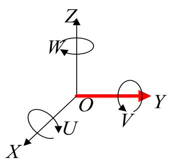
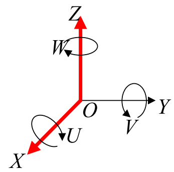
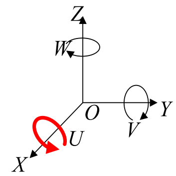
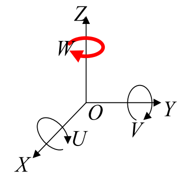
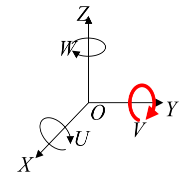
Items	Kinematic error components				
Physical meaning	Positioning error	Straightness error	Pitch	Yaw	Roll
					

Table 2
Kinematic error classifications.

Error term	Parameter	Main Reason
Positioning error	Δy	Power accuracy; assembly error; machining error
Z directional straightness error and pitch error	Δz and Δu	Machining error (guide rail, squareness); assembly error
X directional straightness error and yaw error	Δx and Δw	
Roll error	Δv	Parallelism error

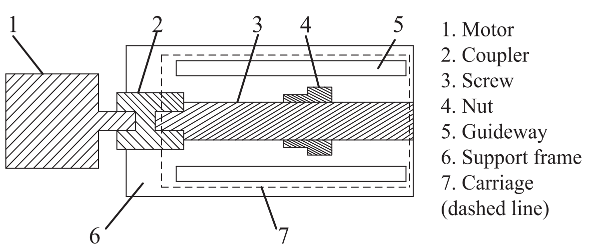


Fig. 2. Schematic for the structure of one PLS.

guideway. Thus, the guideway surface plays a significant role in affecting orientation of carriage, which is straightness and angular errors in PLS.

3.1. Conventional method

Generally, the guideway surface presents a random trend, which contains different characteristics caused by various errors mentioned above. In reality, an arbitrary curve can be represented by a series of Fourier. Similarly, the curve of guideway surface can be formed by a group of Fourier series, as Eq. (1) presents. In order to calculate the value of SaA errors easily and effectively, scholars selected the first term of Fourier series to represent the curve of guideway surface. For example, the author in Ref. [12] demonstrated that the orientation of carriage would be similar to a sinusoidal function by computer simulation, which means the curve of guideway surface is represented by a Fourier function when n=1, see Eq. (2).

$$f(y) = \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \sin\left(\frac{2n\pi y}{\lambda}\right) \tag{1}$$

$$f(y) = \frac{4}{\pi} \sin\left(\frac{2\pi y}{\lambda}\right) \tag{2}$$

By this assumption, this paper carried out a research about the ratio between the length of carriage and period of guideway curve, as Fig. 3 shows, where *f(y)* denotes the guideway surface curve in Z-Y plate, δ denotes vertical straightness error, ε denotes pitch error, λ denotes the wavelength of the given curve *f(y)*, respectively. Based on this assumption, the SaA errors can be calculated straightly at each position by using trigonometric function to represent guideway surface. Some scholars also focused on this subject under this assumption [17,18].

In reality, due to different machining errors, the curve of guide rail is not periodic. Considering the impact by various machining processes, a precise curve is needed to reflect the characteristics of guide rail. In the machining process of guide rail, there are different geometric errors from each manufacturing process, i.e. fine milling/grinding, heat treatment and installation, which lead to various geometric errors. Due to machining process, fine milling/grinding can result in periodic error. In the heat treatment, if the internal force is not totally released, the guide rail will not be straight. Similarly, considered installation error, there is also an arching in the guide rail. The error patterns in different manufacture processes are shown in Fig. 4.

Based on the analysis above, a sinusoidal function is not suitable for representing the curve of guideway surface, which can not reflect the machining characteristic of guide rail. Thus, a systematic and precise method is needed to be raised to obtain accurate expression of guideway surface to calculate the SaA errors.

3.2. SaA errors analysis in new method

Assuming that there is a carriage is sliding along the guide in PLS, and the length is K, as Fig. 5 shows. Before analyzing the relationship between the surface of guideway and SaA errors, some hypotheses are

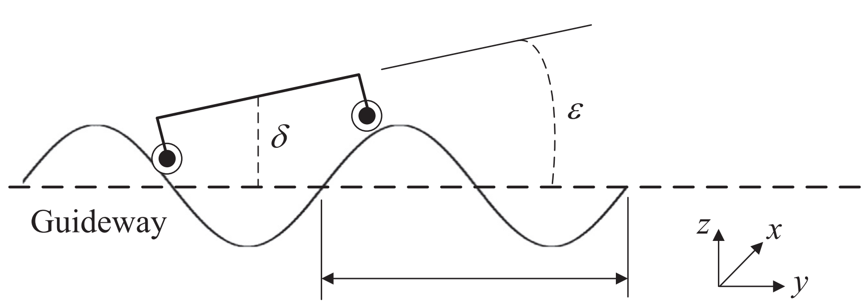


Fig. 3. Schematic for a carriage sliding along guideway in conventional method.

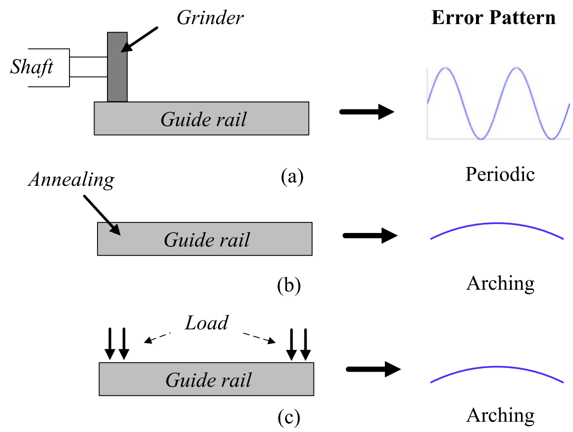


Fig. 4. Schematic for different types of geometric errors in guideway. (a)Fine milling (b) Heat treatment (c) Installation.

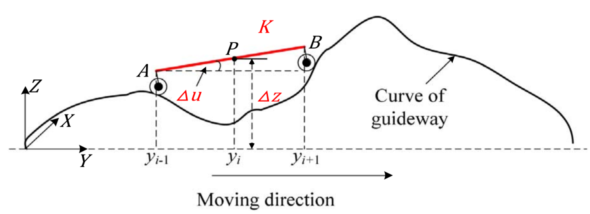


Fig. 5. Schematic for a carriage sliding along a random guideway.

raised first:

a. The contact points between carriage and guideway are regarded as two rigid points *A* and *B* (size is not taken into account).
b. The deformation by load effect and carriage weight etc. is not in consideration.
c. Only quasistatic state is taken into account.
d. Based on Majda [15] and Bryan [19] work, SaA error is represented by the midpoint of carriage (point *P*), the vertical coordinate of *P* Δz denotes the Z-axis straightness,, andΔu denotes the pitch angular error (rotate around X-axis), shown in Fig. 5.

Based on these hypotheses, this study proposes a systematic approach about the relationship between SaA error and curve of guideway surface, and it consists of several steps:

1. Choose precise device to measure guideway surface. A number of papers mentioned about guideway measurement in machine tools, but the guideways in PLS are different due to the size and accuracy magnitude. In industry, Micrometer is widely adopted in guide rail measurement. The geometric tolerance (straightness/parallelism) can be obtained by moving along the guide. However, for majority Micrometer, the accuracy is 0.01 mm to 1 µm, which cannot meet the accuracy requirement. We also considered 3-dimensional measuring device to obtain the guideway surface, such as Confocal Microscope. However, there are some drawbacks. 1. The measuring range is only 1 mm. Thus, a 20 mm guide rail needs 20 times, which increases the measuring error. 2. Due to the characteristic of light focusing in this kind of device, some points have to be compensated by software. Thus, A Digital microscope with 0.01 mm accuracy is adopted in guideway surface measurement.
2. Fit the curve with precise characteristic functions. Considering the real guideway surface curve contains various characteristics, the conventional method by using sinusoidal function is improper to represent it. It is important to adopt corresponding functions to identify these characteristics.
3. Calculate SaA error. Based on the assumptions in Fig. 5 and results from step. 1 and 2, two equations are listed as follows,

$$\frac{y_{i+1} + y_{i-1}}{2} = y_i \tag{3}$$

$$K = \sqrt{[f(y_{i+1}) - f(y_{i-1})]^2 + (y_{i+1} - y_{i-1})^2} \tag{4}$$

where y_i denotes the position calculated in full-path guideway, y_{i-1} and y_{i+1} are the two points *A* and *B* contacting to guide, *f(y)* denotes the fitting curve derived from step. 2, *K* is the length of carriage. The two equations can be used to calculate the coordinates of two contact points if the midpoint coordinate of carriage is y_i. Thus, Z directional straightness error Δz at position y_i and pitch Δu can be obtained by

$$\frac{f(y_{i+1}) + f(y_{i-1})}{2} = \Delta z \tag{5}$$

$$\frac{f(y_{i+1}) - f(y_{i-1})}{K} = \Delta u \tag{6}$$

Therefore, the SaA errors at full guideway can be calculated by following the steps mentioned above.
4. Results comparisons. In order to validate this approach by comparing with the calculated results in step. 3, the SaA errors in PLS need to be measured by laser interferometer. All steps in the new approach are described in Fig. 6:

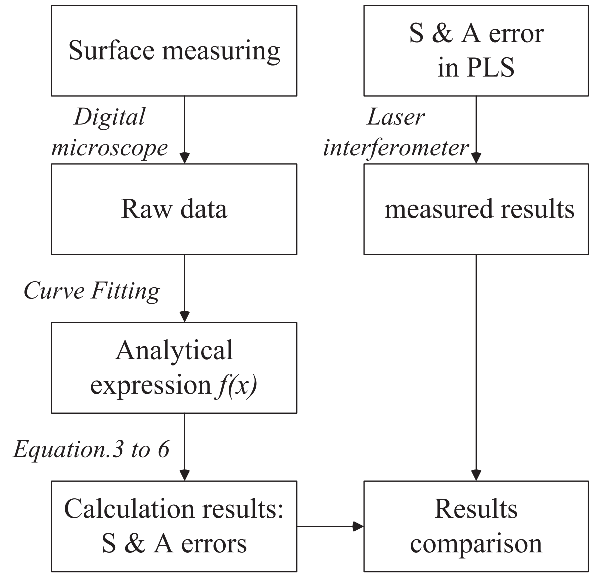


Fig. 6. Flow chart of new approach procedure about S & A errors.

Compared with conventional method, the new approach uses a fitting curve instead of trigonometric function to represent the surface of guideway. This curve in new approach is more accurate than conventional method, which can reflect the characteristic in guideway manufacture process. Due to guideway surface can be measured through precision devices, the guideway measuring and curve fitting in the new approach are more precise than a trigonometric function in conventional work. Based on this approach, through obtaining the guideway surface and other errors, i.e. installation error and machining error of other parts, the SaA error can be calculated in design time. The new approach is beneficial for improving accuracy in error compensation and understanding how the SaA errors be affected in PLS precisely, which is also useful and practical for PLS designer.

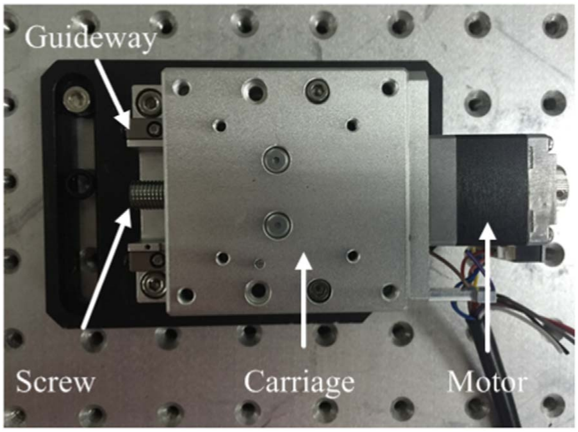


Fig. 7. Structure of a given PLS.

4. Case study

Fig. 7 shows a linear stage widely used in engineering area. It contains a pair of guide rail, lead screw, carriage, motor and other parts. According to Table 2, in this kind of PLS, the positioning error is affected by lead error of screw, the power loss of motor and the assembly error of each part. The SaA errors are affected by the guideway machining error and the assembly error. In order to find how the surface of guideway gives impact on SaA errors in the PLS which is different from previous analysis, a new analyzing approach is employed. All experiments are carried out under constant humidity and temperature environment.

4.1. Guideway measurement and curve fitting

In industrial area, besides Micrometer, engineers usually adopt Dial indicator and Laser autocollimator in guideway measurement. Compared with machine tools, the PLS has some characteristics, such as small size, high-accuracy requirement etc., so the aforementioned measuring devices are not suitable for PLS guideway measurement. In this paper, a Digital microscope with $0.01\mu m$ accuracy is adopted to measure guideway surface. Thus, the procedure of new approach is described as follows.

The stage adopts a pair of cross-roller-guide, shown in Fig. 8(a). The features of this kind of guide rail are low friction, power-loss and elastic deformation, which is widely used in high-accuracy motion system. Fig. 8(b) illustrates the guideway surface experiment measured by Digital microscope. According to Ref. [15], in the high-accuracy-requirement situation, the roller can be regarded as a string considering the stress condition when guideway sliding. Based on the assumptions introduced above, the contact point between roller and guideway can be regarded as the midpoint along Y direction (see Fig. 8(c)), so the line of midpoint along moving direction (Y direction) is considered as the surface of guideway, which needs to be measured. Fig. 8(d) shows the measured curve (scanned by red plate).

Thus, the measured result for guideway surface is shown in Fig. 9.

The characteristics of the guideway are analyzed from Fig. 9. Based on the analysis above, the surface curve is affected by grinding process, assembly error and heat treatment, which reflects two different features. According to Fig. 9, the curve can be determined by two main characteristics. One is comprehensive frequent fluctuation (function $g_1(y)$), matching the machining error, and the other has a longer wavelength (function $g_2(y)$), matching the assembly error and heat treatment, as Fig. 10 shows. Furthermore, there are some peaks and valleys in full-stroke guide because some missing points are compensated in testing by measuring software in Digital microscope.

In consequence, a combination of two trigonometric functions as fitting curve is selected:

$$g_1(y) = a_1\cdot \sin(\omega_1 y + \phi) + b_1\cdot \cos(\omega_1 y + \phi) + C_1;\ g_2(y) = a_2 y^2 + b_2 y + C_2 \tag{7}$$

The fitting function $f_1(y)$ including $g_1(y)$ & $g_2(y)$ is obtained by Eq. (8), and the curve of which is shown in Fig. 11.

$$f(y) = g_1(y) + g_2(y) = a_1\cdot \sin(\omega_1 y + \phi) + b_1\cdot \cos(\omega_1 y + \phi) + a_2 y^2 + b_2 y + C \tag{8}$$

The accuracy comparison in curve fitting for representing guideway is shown in Table 3:

In order to find which function would be precise in representing the guideway surface, the Goodness of Fit (R-square function) is adopted. This function is widely used in linear regression and financial stock, which is considered as an essential way to judge the fitting degree.

In Table 3, Compared with new approach, the accuracy of conventional method is poor in curve fitting to represent guideway surface. These results indicate that a periodic trigonometric function is not suitable for the surface of guideway due to various errors. Consisting of measuring, curve fitting and calculation, the new approach is a systematic and accurate method to study the relationship between guideway and SaA errors. The assumption that the guideway surface can be represented by Eq. (1) if n=1 is beneficial for continuous analysis and calculation of SaA errors. However, majority guideway surfaces are random depending on different machining processes and corresponding deviations. The measurement and related curve fitting method can provide significant results, which also can be applied to other similar environments.

On one hand, the periodic function, like simple Fourier series (n=1), cannot represent different the pattern of guideway surface appropriately. On the other hand, the surface can be denoted by other more accurate polynomials which are related to the errors in machining process. However, it will result in computational efficiency decreasing greatly. This approach links guide rail machining process to guideway surface curve fitting, which is more accurate and practical.

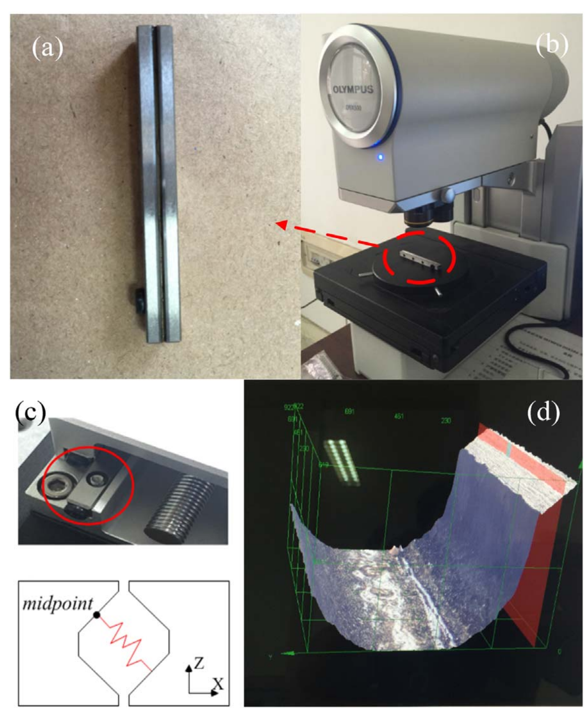


Fig. 8. (a) The cross roller guide (b) Digital Microscope, (c) Schematic for cross roller guide, (d) The measured guideway curve (scanned by plate in red). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

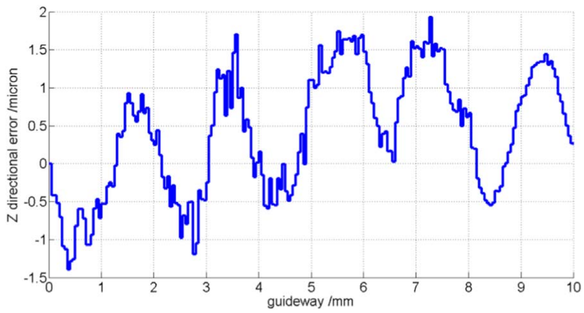


Fig. 9. Surface of guideway measured by digital microscope.

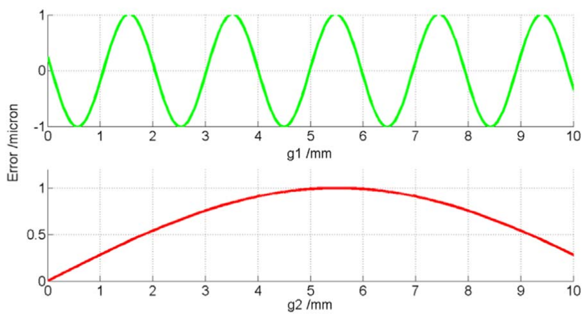


Fig. 10. Characters of measured surface of guideway.

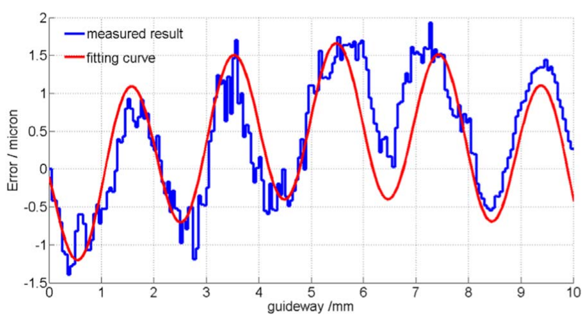
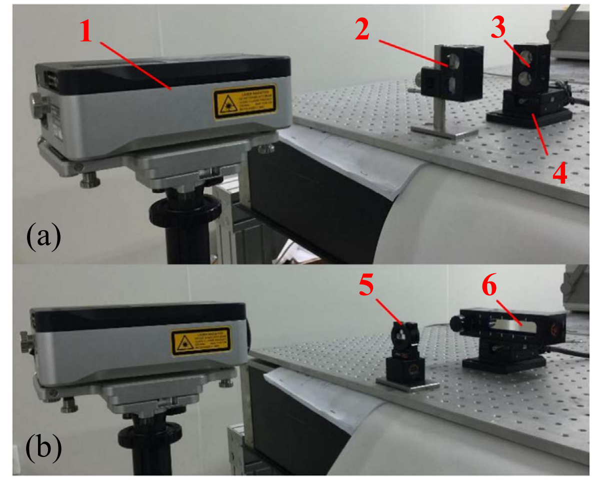


Fig. 11. Comparison between surface measurement and Fitting curve.



1 Laser source 2 Angular interferometer
3 Angular reflector 4 PLS
5 Straightness interferometer 6 Straightness reflector

Fig. 12. The arrangement of SaA error measurement. (a)Angular error (Pitch) (b) Straightness error (Vertical).

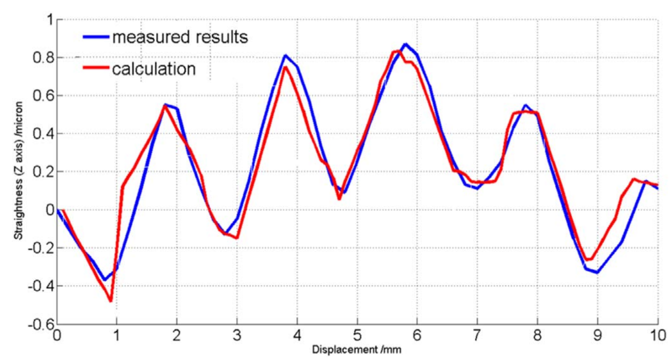


Fig. 13. Results of measured and calculation in straightness in PLS.

4.2. SaA errors calculation and measurement

Based on the fitting curve *f(y)* and corresponding equations about SaA errors, the Z directional straightness $\triangle z_c$ and pitch $\triangle u_c$ can be obtained. In order to prove the validation of the new approach, the SaA errors in linear stage need to be measured simultaneously to compare with SaA error calculation value from Eq. (5) and Eq. (6) based on the fitting curve. Thus, a laser interferometer with 10 nm resolution is utilized to measure the SaA errors in the PLS, as Fig. 12 shows.

The comparison between calculation results and measured results is shown in Fig. 13, respectively.

Fig. 13 illustrates that the calculation results based on the new approach is in agreement with measured curve, such as amplitude, frequency, trend and so on. Considering the theoretical result calculated in ideal situation (some assumptions are introduced in paragraph 3.2), there are some differences existing, especially in peaks or valleys. It is because $\triangle z_c$ and $\triangle u_c$ are theoretical values, and the contacts between carriage and guideway are regarded as rigid points.

The angular errors $\triangle u$ between theoretical and experimental results can also be obtained, as Fig. 14 shows:

The two curves have two characteristics, one is monotonic, and the other is periodic. On one hand, when the carriage slides along guide rail, the angular error $\triangle u_c$ (slope of line AB in Fig. 5) is decreasing gradually due to the arch in guideway surface (function $g_2(y)$). The pitch error $\triangle u_c$ is not an absolute value but a relative value. If the initial value of $\triangle u_c$ is set to 0, the angular error will decrease monotonically, as Fig. 14 shows. On the other hand, the pitch error $\triangle u_c$ fluctuates

Table 3
Comparisons between new method and conventional method in curve fitting.

	New method	Conventional method
Analytical formula	$a_1\cdot\sin(\omega_1 y+\phi)+b_1\cdot\cos(\omega_1 y+\phi)+a_2y^2+b_2y+C$	$f(x)=\frac{4}{\pi}\sum_{n=1,3,5,\cdots}^{\infty}\frac{1}{n}\sin\left(\frac{2n\pi x}{\lambda}\right)$
Parameters values	a_1=0.2475; b_1=−0.9814; w_1=3.199; ϕ =1.25; a_2=−0.031; b_2=0□3636; C=−0.072	n=1;λ=π
Fitting degree (R-square)	0.7047	0.4729

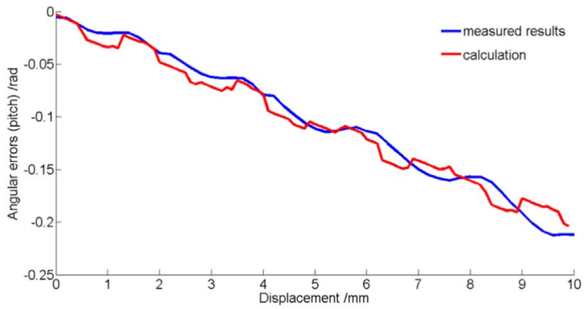


Fig. 14. Results of measured and calculation in pitch in PLS.

periodically because of function $g_1(y)$.

The curve of angular error $\triangle u_c$ calculated by the new approach is in agreement with measured results in amplitude. The guideway surface has a small bump in middle part (according to function $g_2(y)$). In Fig. 14, both theoretical and experimental values generally increase, about 0.2 rad. However, some peaks and valleys are not synchronous. In first 5 mm the trends of two curves are coincide, but in 6 mm~10 mm scale, the calculated result has a phase advance. From the guideway surface measurement in Fig. 11, the fitting curve is synchronous in first 3 valleys (0–5 mm). However, in last 3 valleys (6 mm~10 mm), compared to measured result, the function $f(y)$ cannot fit precisely in amplitude and phase, which resulting in inaccurate results in following calculation.

This paper considers a polynomial with a trigonometric function and a quadratic function to represent the guideway surface, which can reflect the geometric and thermal error of guide rail. However, the comparison between calculated result and measured result indicates that the fitting curve is not precise enough. Thus, the essential part that how to improve the SaA error calculation accuracy of new approach is improving the fitting accuracy, and the cost and computational volume should be considered as well.

Some clauses can be obtained from the analysis above:

1. The surface of the given guideway in measurement has some characteristics, which related to machining processes. One is frequent fluctuation, denoting by g_1, and the other is arching gradually, as g_2 shows. Considering the manufacturing process for guideway, the function g_1 in measurement is related to grinding process, and the function g_2 is related to preload, nut installation and heat treatment.
2. According to the understanding of guideway, two functions g_1 and g_2 are adopted in curve fitting. By using R-square function to weigh the fitting accuracy, the fitting curve in new approach is more accurate than a trigonometric function in conventional method.
3. From the results, it is observed that the range of straightness error in calculation results is from $-0.5\mu m$ to $0.9\ \mu m$, which means the straightness error $\triangle z_c$ is 1.4 μm; and the measured result is from-0.4 μm to 0.9 μm, which means the straightness error $\triangle z_m$ is 1.3 μm. The calculation results of angular error are also in coincidence with the measured result, including the amplitude, frequency and trend. In some peaks and valleys there are few differences between calculation results and measured results. Thus, the fitting accuracy based on guideway surface measurement is essential for following SaA errors calculation in the new approach.

5. Conclusion

This paper introduces a systematic approach on the relationship between surface of guideway and SaA errors. Compared with conventional method using a trigonometric function to represent the surface of guideway, the new approach adopts precise device to measure the surface of guideway and fitting the curve precisely, by which the SaA errors can be calculated with corresponding formulas. Thus, it is precise to know the distribution of SaA errors before installation, which can provide informative guide for engineers and designers. The advantages of the new method are as follows:

1. It is more accurate than conventional method. Previous studies adopted a trigonometric function to represent the surface of guideway, which is not suitable to describe the actual situation. Guideway surface is random and unpredictable due to several manufacture process steps. Thus, the new approach containing measurement and calculation is a precise and systematic thought to obtain SaA errors in design part. In this approach, the surface of guideway is measured first. Then an analytical polynomial with characteristic functions combination is obtained by curve fitting, which reflects the characteristics of guide rail in machining processes. Based on the analytical polynomial, the SaA errors can be calculated through related formulas. Experimental results proved the accuracy and validation of the new method.
2. It is beneficial for designer and engineers. Usually, the SaA error values are needed to be obtained precisely in design part, which is convenient for accuracy improvement. The new approach can provide a precise SaA errors calculation procedure, and it is also related to guideway manufacture process with different characteristics, which is helpful for designer in accuracy control.
3. It is complete and systematic. The analysis procedure of the new method can be applied in different guideway type, which is suitable for other situations. The results indicate that the essential problem of the new approach is the fitting accuracy of the polynomial.

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