Stream-of-Variation (SOV) Theory Applied in Geometric Error Modeling for Six-Axis Motion Platform

Hao Tang, Ji-An Duan, and Shengqiang Lu

***Abstract*—In order to understand how geometric errors propagate and how deviations accumulate in a six-axis motion platform (SMP), a new geometric error model based on the stream of variation (SOV) theory is presented in this paper. SOV theory is widely used in industrial engineering with several steps or phases. The conventional geometric error model only calculates the initial and final orientation, yet the deviations after each step in the whole process are still unknown, which are critical parameters in measuring the product quality. In this new error modeling method, each step in the alignment and welding process in SMP can be considered as a station. Thus, the deviations can also be derived after each station, which is beneficial for error identification. Based on the new error modeling approach involving SOV theory, the validation of an optimized configuration is developed by a series of calculations results. By observing the deviations after each station, the optimized configuration can improve accuracy and reduce power loss compared to a traditional configuration. The new error modeling approach based on SOV theory is systematic and comprehensive, and can be applied in other similar environments.**

***Index Terms*—Configuration optimization, geometric error modeling, laser welding system (LWS), six-axis motion platform (SMP), stream of variation (SOV) theory.**

NOMENCLATURE

LWS	Laser welding system.
SMP	Six-axis motion platform.
OD	Optical device.
SOV	Stream of variation.
X, Y, Z, U, V, W	Six axes in SMP.
x_x^C, y_x^C, z_x^C u_x^C, v_x^C, w_x^C,	Kinematic errors in x-axis.
$\alpha_x^A, \beta_x^A, \gamma_x^A$	Assembly errors in x-axis.
X_z, Y_z, Z_z	Differences between z-axis and frame.
$\boldsymbol{E}$	Transformation matrix.
$\boldsymbol{A}$	Assembly error matrix.
$\boldsymbol{M}$	Movement matrix.
$\boldsymbol{K}$	Kinematic error matrix.
$\boldsymbol{T}$	Deviations of OD.
$\boldsymbol{P}_i$	Ideal coordinate system in station i.
$\boldsymbol{T}_u$	Deviations of upper component.
$\boldsymbol{T}_d$	Deviations of lower component.
$\boldsymbol{T}_D$	Deviations between two components.

Manuscript received April 19, 2016; accepted November 1, 2017. Date of publication December 8, 2017; date of current version February 19, 2020. This work was supported by the National Natural Science Foundation of China under Grant 51705149. This paper was recommended by Associate Editor J. McCall. *(Corresponding author: Hao Tang.)*

The authors are with the College of Mechanical Electronic and Engineering, Hunan University of Science and Technology, Xiangtan 411201, China (e-mail: tanghaocsu@csu.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSMC.2017.2775102

I. INTRODUCTION

SIX-AXIS platform plays an important role in precise manufacturing area [1], [2]. In LWS, two optical components controlled by SMP are aligned under high-accuracy requirements for OD manufacturing [3]. The quality of OD is related to the orientations of two components in the alignment and welding processes, which are affected by the geometric error in SMP. How to observe and calculate the orientation deviations of OD in SMP in the whole manufacturing procedure is critical. Thus, a geometric error model for the SMP applied in LWS is required.

Traditional geometric error modeling method is widely adopted in mechanical engineering regions, i.e., machine tool and coordinate measurement, to evaluate the orientation difference compared to an ideal situation. However, only the initial and final orientations can be collected. In some cases, the initial and final orientations can meet the given accuracy requirements. However, in the middle process, the deviations may be beyond the thresholds, which is unacceptable. For example, in the machining process, it is possible that the initial and final orientations are calculated precisely in the design part. If the deviation of the tool tip in a middle step is beyond the threshold, the part is unqualified. In order to find how the errors are transferring and how deviations are accumulating in the whole procedure, this paper develops a new geometric error modeling method based on SOV theory in SMP. This new error modeling method can improve the accuracy of SMP and the efficiency of OD manufacture.

In order to analyze the impact of the geometric errors on the product quality from the multiaxis system, scholars have studied geometric error modeling and developed some conclusions as guidance. However, few scholars have focused on error modeling in the precise motion platform, though SMP is utilized and plays an essential part in controlling the orientation of components. Compared with large scale equipment, the LWS consists of several small precise motion stages. For error modeling, the majority of the literature is concentrated on three-axis or five-axis machine tools, and some precise systems such as coordinate measuring machines [4]–[6]. In [7], an error model is developed including geometric error and thermal effect by adopting Denavit–Hartenberg transformation. This approach considers all possible errors in a five-axis system, which is complete but complicated. Error modeling, measurement, and compensation in multiaxis machine tools are considered in [8], which was a comprehensive and efficient supplement for error theory. In [9], a five-axis rotary table obtained through adopting the error modeling method is introduced, and a thought is presented that the error model can be divided by the analysis of three translational axes and two rotational axes. However, the formula is too simple in assembly error identification. A geometric error modeling process about misaligned calibration targets in passive microwave remote-sensing systems is developed in [10], which is a typical example of how error modeling approach benefits quality improvement, but this paper only concentrates on machine tools. In [11], a sensitivity analysis based on error modeling in a five-bar-paralleled system is presented, which needs to be adopted in the precise linear stage. The traditional method only focused on the error modeling approach in error compensation. However, the application of error modeling can be broadened, which can provide some innovative thoughts by adopting different knowledge in mathematics and industrial engineering. The classical error modeling approach above was applied to calculate the initial and final orientation in machine tools, but it is not suitable for full-procedure observation. In some cases, how errors transfer and how deviations accumulate should be known because most mechanical systems consist of multistations.

Another group of scholars had been working on the optimization for the general error modeling approach. The work in [12] introduced geometrical error modeling method by using neural networks in two-axis motion stage. In this approach, the error components can be obtained through a neural networks learning process. However, the calculation results are approximate values, which is not suitable for high-accuracy requirements. In [13], a general approach is presented for error modeling of machine tools. This approach divided the traditional model into two separations, which can provide informative guidelines for engineers for accuracy improvement. In [14], a classical example of error modeling is given for a five-axis machine tool based on the matrix summation approach. This method reduced the computational volume compared to the traditional homogeneous transformation matrix method. However, the approach is only suitable for five-axis machine tools, which needs to be applied to other configurations. In [15], a new geometric error modeling method based on SOV theory is introduced, which is adopted in the optoelectronic packaging system. This paper raised some new concepts in error model, such as using "square" to measure deviation. Nevertheless, those approaches mentioned above have some drawbacks. Only the initial and final orientation of the object is observed, yet the trend of errors in whole process cannot be obtained. The potential risk based on conventional error modeling method is that the error value has the possibility to exceed threshold or accuracy requirement in the middle step. In LWS, it is necessary to observe whether the optical power meets the requirement, which affects alignment accuracy and manufacturing efficiency. Therefore, how errors propagate and how deviations accumulate in the whole procedure in LWS is critical for device quality, and an optimal geometric error model that can observe the deviation trend is necessary.

In order to solve these problems, a new error model based on the SOV theory is introduced in this paper. The SOV theory is common in industrial engineering area due to complicated mechanical structure and process, such as manufacturing and machining, information transferring, and service processing. For these considerations, some scholars concentrated on this topic in order to reduce system error and improve efficiency [16]–[19]. In [18], SOV theory is adopted in error measurement in multistation machining systems. In most cases, the quality of final product in a multistation system is determined by a series of steps. The deviations of product are affected by the errors generated in the current step, and the accumulated errors transmitted from previous stations. Unlike calculating deviations in the final step (for judging the product qualified or unqualified), this approach can reveal the errors from each station and how deviations accumulate after each station throughout the whole procedure. In order to improve accuracy and quality of OD in the whole procedure, the SOV theory is utilized in geometric error modeling in this paper. The steps in laser welding procedure can be regarded as individual stations, and the power loss of OD can be regarded as product quality. By adopting the new error modeling approach, it is easy to understand how much optical power is lost in the laser welding procedure, which is not only helpful to improve accuracy in SMP in LWS but also improve efficiency in OD manufacturing.

This paper is organized as follows. Section II introduces a classical configuration of SMP and the OD manufacturing steps in LWS, and the power loss of OD at different directions are also presented. Section III introduces the definitions of geometric errors and conventional error modeling approach. Section IV briefly introduces the SOV theory and presents the new geometric error modeling method based on SOV theory. The derivation and definition mapping between geometric error modeling and SOV theory prove the validation of this new method. A case of LWS is carried out in Section V by employing the new modeling method, and the results indicate that the new method is helpful to select an optimal configuration compare with conventional error modeling method. Section VI as a conclusion shows the advantages of the new geometric error modeling method by involving SOV theory.

II. Six-Axis Platform in LWS

A typical LWS structure is shown in Fig. 1. It consists of SMP, adapter, alignment unit (upper and lower components), welding units, optical power feedback equipment, and frame. Three translational axes are named x, y, and z axes, and three rotational axes are named u, v, and w axes, respectively. The upper component and lower component are aligned through precise movements of SMP.

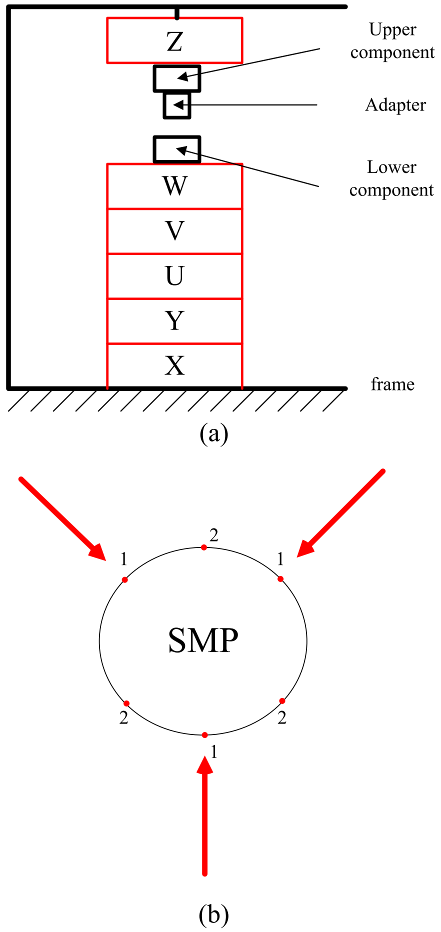


Fig. 1. (a) Typical configuration of SMP used in LWS. (b) Schematic of welding process including welding step.

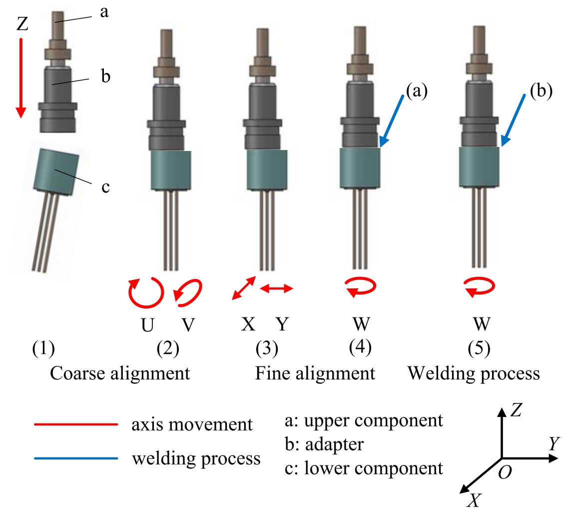


Fig. 2. OD manufacture procedure.

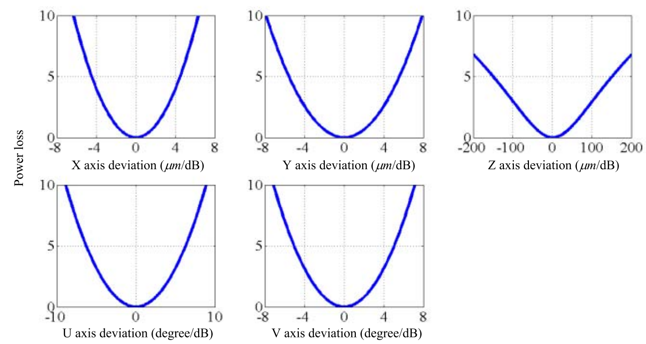


Fig. 3. Relationship between different deviations and power loss.

The manufacture process for OD can be divided into several steps.

1) *First Step:* Coarse alignment:
   a) The two components close (z-axis moves).
   b) Find the aligning plates of the two components (u and v axes rotate).
2) *Second Step:* Fine alignment:
   a) Position adjustment (x- and y-axes move).
   b) Orientation adjustment (w-axis rotates).
3) *Third Step:* Welding process:
   a) First welding.
   b) w-axis rotates.
   c) Second welding.

All steps are shown in Fig. 2.

The diagram above indicates that the LWS procedure is complicated. Various reasons undermined the quality of OD in each step, including the geometric error in SMP in alignment and the quality of laser beam in the welding process, etc. Under idea environment (constant temperature and humidity), the geometric error of platform overwhelms other factors. Thus, an error model for the SMP which is applied in LWS is required. In this paper, geometric errors are taken into account in error modeling; other errors, such as stability of laser beam, post-weld shift, etc., are not in consideration.

In order to guarantee high-accuracy, low power loss, and high-efficiency requirements, the deviations in alignment should be analyzed. Different directional deviations result in different power loss. Based on the research in [20], the alignment between two components in LWS is shown in Fig. 3.

The results in Fig. 3 point out that x and y directional deviation are sensitive to the quality of OD, u and v directional deviation are relative sensitive, and z directional deviation is insensitive.

III. Geometric Errors and Conventional Approach

Generally, for a motion axis, there are six directional deviations in movement, three translational errors, named x, y, and z, and three rotational errors, named α, β, and γ. Fig. 4 shows a translational axis moving along x-axis and a rotational axis rotating around z-axis.

In Fig. 4, in a translational axis, the positioning error is x_x^C, the two straightness errors are y_x^C and z_x^C, and pitch, yaw, and roll errors are γ_x^C, α_x^C, and β_x^C, respectively. In a rotational axis, the angular error is γ_w^C (similar to the positioning error in translational axis), two tilt errors are α_w^C and β_w^C, and the axial error and two radius errors are z_w^C, x_w^C, and y_w^C. All kinematic errors in other directions listed in Table I.

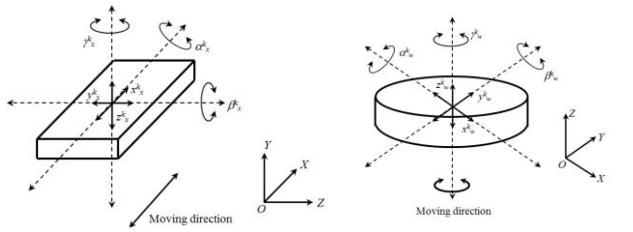


Fig. 4. Schematic of a translational axis and rotational axis with six kinematic errors.

TABLE I
PHYSICAL MEANING OF KINEMATIC ERRORS AT SIX AXES

Axis	Error components				
	Positioning error	Straightness error	Pitch	Yaw	Roll
X-axis	$x^C{}_x$	$y^C{}_x, z^C{}_x$	$\beta^C{}_x$	$\gamma^C{}_x$	$\alpha^C{}_x$
Y-axis	$y^C{}_y$	$x^C{}_y, z^C{}_y$	$\gamma^C{}_y$	$\alpha^C{}_y$	$\beta^C{}_y$
Z-axis	$z^C{}_z$	$x^C{}_z, y^C{}_z$	$\alpha^C{}_z$	$\beta^C{}_z$	$\gamma^C{}_z$
	Axial error	Radius error	Angular error	Tilt error	
U-axis	$x^C{}_u$	$y^C{}_u, z^C{}_u$	$\alpha^C{}_u$	$\beta^C{}_u, \gamma^C{}_u$	
V-axis	$y^C{}_v$	$x^C{}_v, z^C{}_v$	$\beta^C{}_v$	$\alpha^C{}_v, \gamma^C{}_v$	
W-axis	$z^C{}_w$	$x^C{}_w, y^C{}_w$	$\gamma^C{}_w$	$\alpha^C{}_w, \beta^C{}_w$	

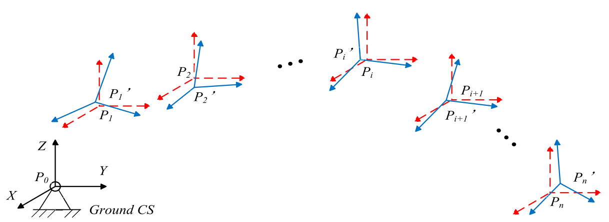


Fig. 5. Schematic of a multibody system denoting by ideal and actual coordinate systems.

The assembly error, resulting from imperfect installation between two units, exists between two axes. In most cases, the assembly errors concentrate on angular errors, i.e., verticality or parallelism.

The conventional error modeling method is deriving the pose of object in ideal and actual states, and calculating the difference between the two states. For example, there is a multibody system in Fig. 5. The red coordinate system denotes the ideal state for each body using P_1 to P_n and the blue coordinate system denotes the actual situation, using P' to P'_n.

In order to calculate how much effect the errors give in this system, kinematic errors, and assembly errors should be into consideration. If transformation matrix $\boldsymbol{E}$ adopted to denote the difference between two adjacent coordinate systems, the highest body P_n can be derived as

$$P'_n = {}^0\boldsymbol{E}_n \cdot P_0 \tag{1}$$

where

$$ {}^0\boldsymbol{E}_n = {}^0\boldsymbol{E}^1_1\boldsymbol{E}_2 \cdots {}^{i-1}\boldsymbol{E}^i_i\boldsymbol{E}_{i+1} \cdots {}^{n-1}\boldsymbol{E}_n \tag{2}$$

where ${}^{i-1}\boldsymbol{E}_i$ denotes transformation matrix from P_i to P_{i-1}.

In an ideal situation, the pose of the object $\boldsymbol{T}_0$ in coordinate can be derived

$$\boldsymbol{T}_0 = {}^0\boldsymbol{E}_n\boldsymbol{T}_n. \tag{3}$$

If the errors are considered, the actual pose $\boldsymbol{T}'_0$ is expressed as follows:

$$\boldsymbol{T}'_0 = {}^0\boldsymbol{E}_n\boldsymbol{T}'_n. \tag{4}$$

Thus, using actual pose $\boldsymbol{T}'_0$ to subtract ideal pose $\boldsymbol{T}_0$, the deviation can be calculated as

$$\boldsymbol{D}_n = \boldsymbol{T}'_0 - \boldsymbol{T}_0. \tag{5}$$

$\boldsymbol{D_n}$ denotes the deviations in actual status.

The above procedure of conventional error modeling method has some disadvantages.

1) Difficult to know error propagation procedure.
2) Difficult to debug.
3) Can only be applied to limited applications in most cases. Usually, for a given multiaxis system, the error model is established for error prediction and compensation in design part. In some cases, it is necessary to expand the application range, i.e., configuration selecting and sensitive step control in certain environment.

IV. Stream of Variation Theory and Error Modeling

SMP is used for controlling the orientation of two components in OD alignment. The conventional method only calculated the pose and orientation of OD initially and finally. Through calculating the difference between initial state and final state, the deviations can be derived. However, how the deviations propagate after each step mentioned above is important, which is related to power loss and quality control. If a low power loss state is kept in whole laser welding procedure, the aligning accuracy and manufacture efficiency of OD can be guaranteed. Thus, a comprehensive geometric error model for SMP used in OD manufacture in LWS is needed, which is also helpful for continued error compensation in platform accuracy improvement.

A. *Stream of Variation Theory Methodology*

This paper adopts SOV theory into optimizing error modeling methodology. SOV theory is widely used in engineering, i.e., complicated manufacture and information communication [16], which consists of multiple phases and steps required to fabricate a product in the final phase. In order to guarantee the power loss not beyond the threshold, the OD orientation should be controlled strictly during the alignment and welding process in LWS, so how the deviations fluctuate in the full procedure should be known. Therefore, SOV theory is employed in LWS to solve the problem.

An illustrative schematic of SOV theory is shown in Fig. 6.

The whole system in Fig. 6 is grouped by N stations. A classical formula is helpful in understanding SOV theory

$$\boldsymbol{T}_i = \boldsymbol{B}_{i-1}\boldsymbol{T}_{i-1} + \boldsymbol{R}_i\boldsymbol{x}_i + \boldsymbol{w}_i \tag{6}$$

where $\boldsymbol{T}_0$ denotes the initial state of product quality, $\boldsymbol{T}_i$ denotes the state after station i, $\boldsymbol{T}_{i-1}$ denotes the upstream effect from previous stations, $\boldsymbol{x}_i$ denotes the error effect resulting from station i, $\boldsymbol{w}_i$ denotes measured errors in x_i, $\boldsymbol{B}_i$, and $\boldsymbol{R}_i$ are corresponding coefficient for $\boldsymbol{T}_i$ and $\boldsymbol{x}_i$ in matrix form. The

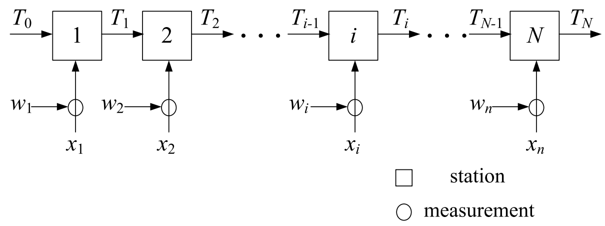


Fig. 6. Schematic of SOV theory.

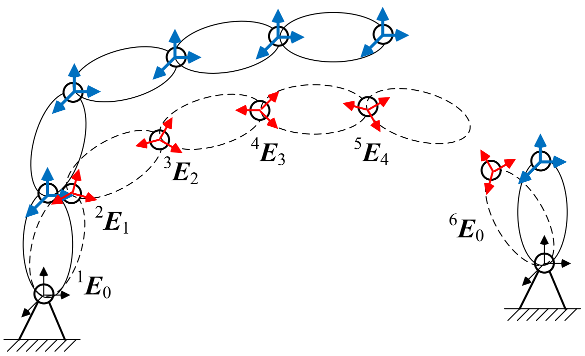


Fig. 7. Schematic of kinematic chain for SMP in LWS.

diagram can calculate the deviations after every station, and describe how the product quality changes and how deviations propagate in the whole system.

On this basis, the SOV theory can be adopted into error modeling for SMP in LWS. Each step in Fig. 2 is regarded as a station, and the calculated deviations after each station are considered as upstream impact on the next station until the final one. Therefore, it is beneficial for calculating how error transfers and how optical power loses in whole procedure.

B. New Error Modeling Approach

In the analysis of error modeling, a kinematic chain is established to derive the pose of an object in the coordinate system in Fig. 7, which is helpful in understanding how the errors transferred from the lower coordinate system to the higher one.

The geometric error consists of kinematic error and assembly error. Kinematic errors should be considered because each coordinate system is mobile, and assembly error exists between two adjacent axes due to nonstrict-orthogonality. Because the deviations can be derived through the difference between the actual state and the ideal state, a transformation matrix $\boldsymbol{E}$ is adopted to calculate the deviations. The transformation matrix $\boldsymbol{E}$ includes assembly error matrix $\boldsymbol{A}$, movement matrix $\boldsymbol{M}$, and kinematic error matrix $\boldsymbol{K}$. The multiplication order needs to be highlighted because matrix multiplication is not suitable for the Commutative Law. Based on the work in [21], it is inferred that the motion error matrix should postmultiplied by the error matrix, and the assembly error should be premultiplied by the error matrix. This is because the motion error matrix is generated in motion stage, and assembly error existed before axis movement. All error matrices are shown in Table II.

TABLE II
HTM OF GEOMETRIC ERROR MATRIX

Error terms	Error matrix
Assembly error matrix	$\boldsymbol{A}_x = \begin{bmatrix} 1 & -\gamma_x^A & \beta_x^A & 0 \\ \gamma_x^A & 1 & -\alpha_x^A & 0 \\ -\beta_x^A & \alpha_x^A & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Kinematic error matrix	$\boldsymbol{K}_x = \begin{bmatrix} 1 & -\gamma_x^C & \beta_x^C & x_x^C \\ \gamma_x^C & 1 & -\alpha_x^C & y_x^C \\ -\beta_x^C & \alpha_x^C & 1 & z_x^C \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Note: the superscript A & C denotes the error terms, and the subscript x denotes which axis belongs to. Other directional geometric error matrices can be derived similarly.

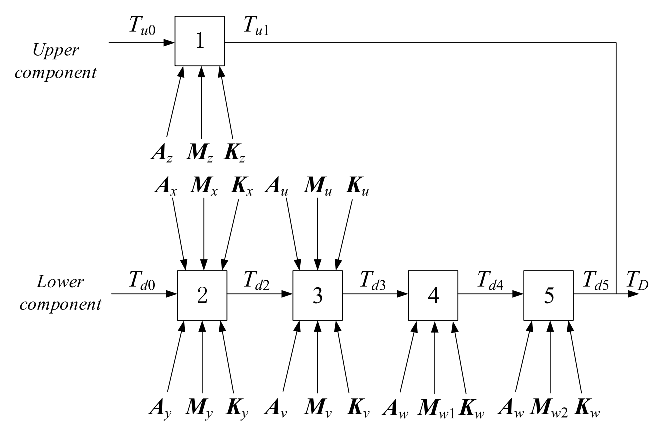


Fig. 8. Schematic of an error model based on SOV theory in LWS.

Thus, in Fig. 7, the deviations of OD in LWS can be calculated by

$$\boldsymbol{T} = {}^5\boldsymbol{E}_0\boldsymbol{P}_1 - {}^6\boldsymbol{E}_0\boldsymbol{P}_2. \tag{7}$$

$\boldsymbol{P}_i$ denotes the ideal coordinate system in station i. The former part ${}^5\boldsymbol{E}_0\boldsymbol{P}_1$ is the orientation of the upper component after considering geometric error, and the latter part ${}^6\boldsymbol{E}_0\boldsymbol{P}_2$ is calculating the orientation of the lower component

$$\begin{aligned} {}^5\boldsymbol{E}_0 &= {}^1\boldsymbol{E}_0{}^2\boldsymbol{E}_1{}^3\boldsymbol{E}_2{}^4\boldsymbol{E}_3{}^5\boldsymbol{E}_4 \\ &= \boldsymbol{A}_x\boldsymbol{M}_x\boldsymbol{K}_x\boldsymbol{A}_y\boldsymbol{M}_y\boldsymbol{K}_y\boldsymbol{A}_u\boldsymbol{M}_u\boldsymbol{K}_u\boldsymbol{A}_v\boldsymbol{M}_v\boldsymbol{K}_v\boldsymbol{A}_w\boldsymbol{M}_w\boldsymbol{K}_w \end{aligned} \tag{8}$$

$${}^6\boldsymbol{E}_0 = \boldsymbol{A}_z\boldsymbol{M}_z\boldsymbol{K}_z. \tag{9}$$

The subscript denotes to the moving direction. All matrices are four-order matrix. The first three-column stands for rotational deviations, and the fourth column stands for translational deviations. Conventional method reveals how the orientation of object is affected by different error sources, but the error propagation and deviation accumulation are still unclear.

In order to develop the new error modeling approach based on the SOV theory, the concepts in error modeling need to be mapped to the SOV theory. The steps in LWS can be regarded as stations, the geometric errors in alignment can be regarded as error source, the deviations of OD can be regarded as product quality, and the deviations from previous stations can be regarded as upstream impact, respectively.

In the new error modeling approach based on the SOV theory, how errors propagate and how optical power loses during the whole procedure can be illustrated. On this basis, the movements of SMP in the flow chart in Fig. 2 can be transferred into SOV form (welding steps are not shown), as is shown in Fig. 8.

For example, the deviation matrix $\boldsymbol{T}_{u1}$ that denotes the orientation of upper component after step 1) is shown as follows:

$$\begin{aligned}\boldsymbol{T}_{u1} &= \boldsymbol{A}_z\boldsymbol{M}_z\boldsymbol{K}_z\boldsymbol{P}_{u0} - \boldsymbol{M}_z\boldsymbol{P}_{u0}\\ &= \begin{bmatrix} 1 & -\gamma_z^A & \beta_z^A & 0\\ \gamma_z^A & 1 & -\alpha_z^A & 0\\ -\beta_z^A & \alpha_z^A & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & X_z\\ 0 & 1 & 0 & Y_z\\ 0 & 0 & 1 & Z_z\\ 0 & 0 & 0 & 1\end{bmatrix}\\ &\quad\times \begin{bmatrix} 1 & -\gamma_x^C & \beta_x^C & x_x^C\\ \gamma_x^C & 1 & -\alpha_x^C & y_x^C\\ -\beta_x^C & \alpha_x^C & 1 & z_x^C\\ 0 & 0 & 0 & 1\end{bmatrix} \cdot \boldsymbol{P}_{u0}\\ &\quad- \begin{bmatrix} 1 & 0 & 0 & X_z\\ 0 & 1 & 0 & Y_z\\ 0 & 0 & 1 & Z_z\\ 0 & 0 & 0 & 1\end{bmatrix} \cdot \boldsymbol{P}_{u0}.\end{aligned} \tag{10}$$

The former part $\boldsymbol{A}_z\boldsymbol{M}_z\boldsymbol{K}_z\boldsymbol{P}_{u0}$ denotes actual orientation, and the latter part $\boldsymbol{M}_z\boldsymbol{P}_{u0}$ denotes ideal orientation without geometric errors. X_z, Y_z, and Z_z denote the differences between z-axis coordinate system and frame, $\boldsymbol{T}_{u1}$ contains six directional deviations, and it can be also transferred into relationship between power loss and deviations.

Similarly, the deviations of lower component after stations 2 to 5 can be calculated

$$\boldsymbol{T}_{d2} = \boldsymbol{A}_x\boldsymbol{M}_x\boldsymbol{K}_x\boldsymbol{A}_y\boldsymbol{M}_y\boldsymbol{K}_y\boldsymbol{P}_{d0} - \boldsymbol{M}_x\boldsymbol{M}_y\boldsymbol{P}_{d0} \tag{11}$$

and the deviation $\boldsymbol{T}_{d2}$ is considered as upstream impact on next step

$$\boldsymbol{T}_{d3} = (\boldsymbol{T}_{d2} + \boldsymbol{P}_{d2})\boldsymbol{A}_u\boldsymbol{M}_u\boldsymbol{K}_u\boldsymbol{A}_v\boldsymbol{M}_v\boldsymbol{K}_v\boldsymbol{P}_{d2} - \boldsymbol{M}_u\boldsymbol{M}_v\boldsymbol{P}_{d2} \tag{12}$$

$$\boldsymbol{T}_{d4} = (\boldsymbol{T}_{d3} + \boldsymbol{P}_{d3})\boldsymbol{A}_w\boldsymbol{M}_{w1}\boldsymbol{K}_w\boldsymbol{P}_{d3} - \boldsymbol{M}_{w1}\boldsymbol{P}_{d3} \tag{13}$$

$$\boldsymbol{T}_{d5} = (\mathrm{T}_{d4} + \boldsymbol{P}_{d4})\boldsymbol{A}_w\boldsymbol{M}_{w2}\boldsymbol{K}_w\boldsymbol{P}_{d4} - \boldsymbol{M}_{w2}\boldsymbol{P}_{d4}. \tag{14}$$

Thus, the deviation between two components in alignment is $\boldsymbol{T}_D$

$$\boldsymbol{T}_D = \boldsymbol{T}_{d5} - \boldsymbol{T}_{u1}. \tag{15}$$

From the work in [15], we can know that the accuracy of two error modeling approaches is in the same magnitude. The new error model based on SOV theory has the following advantages.

1) It is easy to understand how errors accumulate, which can be used to control product quality precisely.
2) It is easy to debug, because the error modeling procedure is in agreement with the working process.
3) It is beneficial for continued applications, such as sensitive error control and configuration optimization.

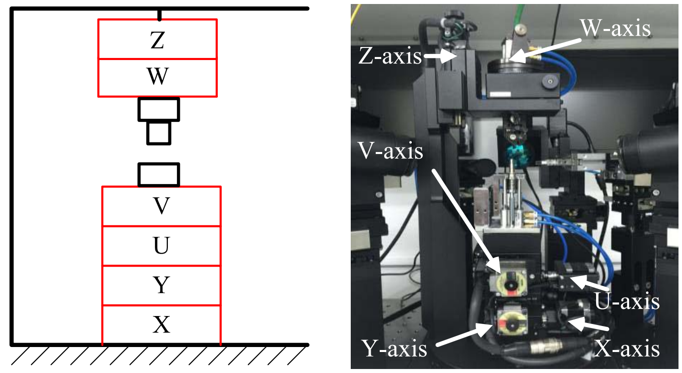


Fig. 9. New configuration of SMP in LWS.

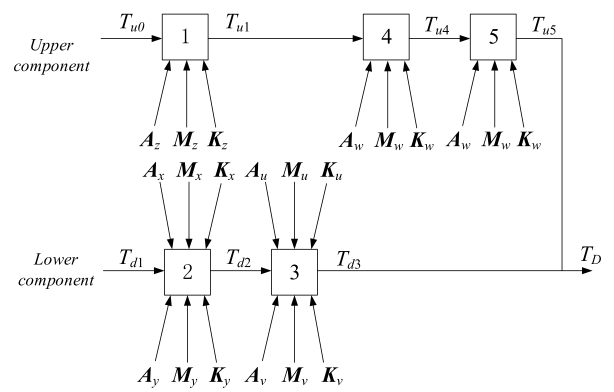


Fig. 10. Schematic of a new error model based on SOV theory in LWS.

TABLE III
VALUE OF MOVEMENTS AND ERROR TERMS

Movement	X, Y, Z axes movements	U, V, W axes movements	
Value	1 mm	1°	
Error term	Translational errors	Rotational error	Assembly errors
Value	1 μm	0.1°	0.5°

V. CONFIGURATION OPTIMIZATION

There are several configurations of LWS in industrial working area, such as *F-Z-W-Y-X-V-U*, *X-Y-Z-F-U-V-W*, etc. Consider the effects caused by rotational axis, an optimal configuration, which is not only helpful in accuracy and error compensation but also beneficial for efficiency improvement is necessary. How to choose an optimal configuration for the SMP in LWS is important. On this basis, another configuration is raised to compare with the traditional one, as Fig. 9 shows.

The orientation of the upper component is affected by the w-axis in the configuration shown in Fig. 9. Based on the analysis above, the configuration can be transferred into SOV theory form (similar to Fig. 6), as is shown in Fig. 10.

In order to know which configuration is optimal in LWS, a series of values are given for all errors and movement in the worstcase to calculate the deviations of OD, as is presented in Table III. First, the deviation results of the two configurations by conventional error modeling method are calculated in Table IV.

TABLE IV
FINAL DEVIATIONS BETWEEN TWO CONFIGURATIONS
(“**F**” DENOTES “FRAME”)

Configuration	Deviation matrix
Z-**F**-X-Y-U-V-W	$D=\begin{bmatrix} 1 & -0.0020 & 0.0030 & 0.0049 \\ 0.0020 & 1 & -0.0030 & 0.0050 \\ -0.0030 & 0.0030 & 1 & 0.0056 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
W-Z-**F**-X-Y-U-V	$D=\begin{bmatrix} 1 & -0.0004 & 0.0006 & 0.0009 \\ 0.0004 & 1 & -0.0006 & 0.0010 \\ -0.0006 & 0.0006 & 1 & 0.0010 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The results indicate that the second configuration has less deviation in six directions, which means it is more suitable as SMP in LWS.

For the majority of multiaxis systems, the conventional method can be adopted to observe the final deviations. However, in some specified situation, the orientation control in whole procedure is important. For instance, in a CNC machine, if there is a displacement threshold for the tool tip position, like over-machining control, the requirement can hardly be met by a conventional error modeling approach because it is difficult to know the orientation of the tool tip in the procedure. Thus, it is necessary to understand how errors propagate and how deviations accumulate, which needs the new error modeling approach based on SOV theory.

The x and y directional deviations are sensitive errors in LWS, as mentioned in Section III. Similarly, the two configurations are analyzed by the new error modeling approach. In order to observe the deviations in the whole procedure in two configurations, the six directional deviations are calculated after each step.

The x directional deviation is considered as an example, as shown in Fig. 11. The x directional deviation is decreased from 4.9 μm to 0.9 μm. Other directional deviations can be calculated with the same procedure where the blue line and red line denote x directional deviation of the upper and lower component after each station, respectively. It also can be mapped to flow charts in SOV form like in Figs. 6 and 8. Compared with the conventional configuration (*Z-F-X-Y-U-V-W*), the x directional deviation is reduced from 4.9 μm to 0.9 μm. Thus, how x directional deviations accumulated can be observed obviously.

In order to know the deviation of OD, a subtraction between the deviations of the upper component and lower component by the results in Fig. 11 can be calculated. Fig. 12 shows x directional deviation of OD after the steps.

In the first three stations, there is no difference between the two configurations from Fig. 12 because of the same procedure. However, after station 3, x directional deviation decreases in the new configuration, which is opposite in traditional configuration.

The new error modeling approach-based SOV theory also can be adopted for sensitive error term control. In Fig. 12(b), there is a peak after station 3, so this step (u and v axes rotate) should be acknowledged. However, a conventional error modeling method that calculates initial and final orientation cannot finish the task.

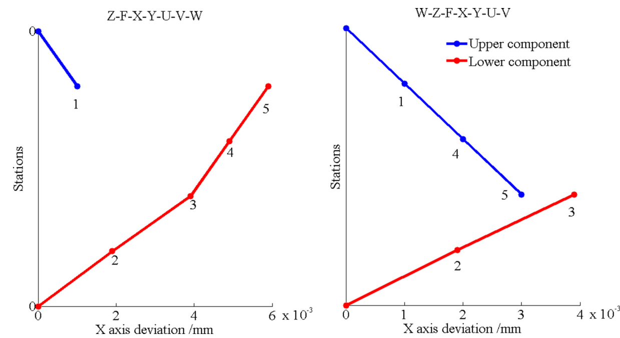


Fig. 11. x directional deviations of upper and lower components in two configurations.

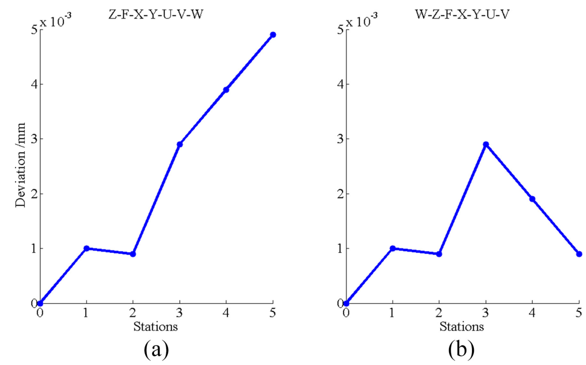


Fig. 12. x directional deviations of OD between two different configurations.

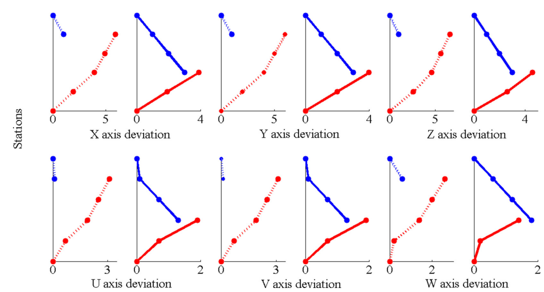


Fig. 13. Six directional deviations of upper and lower components between two configurations.

Consequently, other directional deviations can be derived through the same procedure shown in Fig. 13. The red line represents the deviation in lower component, and the blue line represents the deviations in upper component. The full line and dashed line represent the deviations in optimized and traditional configurations, respectively. The results are shown in Table V.

In Fig. 13, the dashed line stands for traditional configuration, and the full line stands for optimized configuration, which has less deviation in six directions. Similarly, the trend of OD deviations is shown in Fig. 14. The full line and dashed line represent the deviations in optimized and traditional configurations, respectively. The units at translational and rotational direction are μm and *mrad*.

As predicted above, the deviations of OD in the optimized configuration are reduced.

TABLE V
RESULTS ABOUT SIX DIRECTIONAL DEVIATIONS AFTER STATION 5 BETWEEN TWO CONFIGURATIONS

Axis deviation	Traditional configuration	Optimized configuration
X (mm)	0.0049	0.0009
Y (mm)	0.0050	0.0010
Z (mm)	0.0056	0.0010
U (rad)	0.0030	0.0006
V (rad)	0.0030	0.0006
W (rad)	0.0020	0.0004

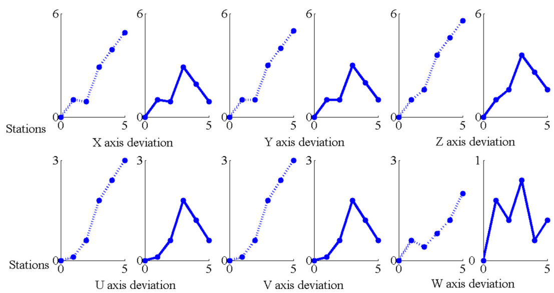


Fig. 14. Comparison between two configurations in six directional deviations of OD.

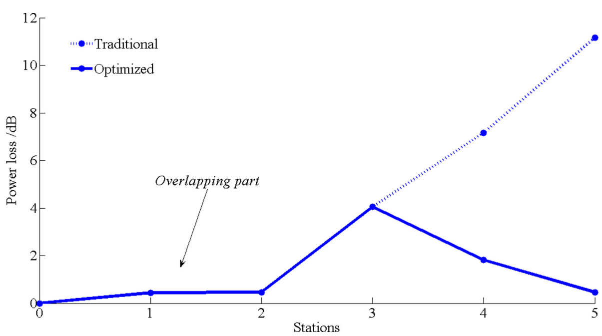


Fig. 15. Comparison between two configurations in OD power loss.

Furthermore, based on the conclusion in Section II, the five directional deviations can impact the power loss of OD. Thus, the variation of power loss in two configurations is illustrated in Fig. 15.

From the analysis above, the optimized configuration is optimal in LWS, not only in final step but also decreasing the deviations in the whole procedure, which cannot be observed in the conventional method. Although the final deviation may be good in some situations, it is possible that there will be a great spike or gap in the middle of the whole system, which is unacceptable. In order to solve the problem, the new error modeling approach based on SOV theory is developed to find how deviations accumulate and how errors propagate. The new approach is helpful in optimal configuration selection, and also in sensitive error control. Results indicate that the optimized configuration *W-Z-F-X-Y-U-V* is more suitable for the laser welding procedure.

VI. CONCLUSION

This paper introduces a new error modeling approach based on SOV theory in LWS. Unlike conventional the error modeling method that only calculates the initial and final orientation of OD, the new approach evaluates the steps in LWS as stations, and calculates the deviations after each station. It is helpful in understanding how errors propagate and how deviations accumulate, which can be adopted in configuration optimization and sensitive error control. Because it is important to observe the power loss in the laser welding procedure, the new approach is not only beneficial for error compensation but also for contributing to aligning efficiency improvement. The new error modeling approach based on the SOV theory has some advantages, as follows.

1) Calculating the deviations of OD after each station in LWS, which is helpful for understanding the trend of power loss in the whole laser welding procedure.
2) Selecting an optimal configuration for LWS. By calculating the deviations of two components in each station, the deviation of OD and the power loss can be calculated precisely, which can improve accuracy and aligning efficiency. It also can be used for sensitive error control.
3) It is systematic and comprehensive. The deviations analysis for OD based on SOV theory can be used to similar environments and systems.

REFERENCES

[1] I. Sharp, K. Yu, and T. Sathyan, “Positional accuracy measurement and error modeling for mobile tracking,” *IEEE Trans. Mobile Comput.*, vol. 11, no. 6, pp. 1021–1032, Jun. 2012.
[2] M. Fadhali, J. Zainal, Y. Munajat, J. Ali, and R. A. Rahman, “Efficient coupling and relaxed alignment tolerances in pigtailing of a laser diode using dual ball lenses,” *Optik Int. J. Light Electron Opt.*, vol. 120, no. 8, pp. 384–389, 2009.
[3] Y.-C. Hsu, J.-H. Kuang, Y.-C. Tsai, and W.-H. Cheng, “Investigation and comparison of postweld-shift compensation technique in TO-Can- and butterfly-type laser-welded laser module packages,” *IEEE J. Sel. Topics Quantum Electron.*, vol. 12, no. 5, pp. 961–969, Sep./Oct. 2006.
[4] A. C. Okafor and Y. M. Ertekin, “Derivation of machine tool error models and error compensation procedure for three axes vertical machining center using rigid body kinematics,” *Int. J. Adv. Mach. Tools Manuf.*, vol. 40, no. 8, pp. 1199–1213, 2000.
[5] S. W. Zhu *et al.*, “Integrated geometric error modeling, identification and compensation of CNC machine tools,” *Int. J. Mach. Tools Manuf.*, vol. 52, no. 1, pp. 24–29, 2012.
[6] L. C. Hale, “Principles and techniques for designing precision machines,” Ph.D. dissertation, Lawrence Livermore Nat. Lab., Univ. California, Livermore, CA, USA, 1999.
[7] A. K. Srivastava, S. C. Veldhuis, and M. A. Elbestawi, “Modelling geometric and thermal errors in a five-axis CNC machine tool,” *Int. J. Mach. Tools Manuf.*, vol. 35, no. 9, pp. 1321–1337, 1995.
[8] M. Rahman, J. Heikkala, and K. Lappalainen, “Modeling, measurement and error compensation of multi-axis machine tools. Part I: Theory,” *Int. J. Mach. Tools Manuf.*, vol. 40, no. 10, pp. 1535–1546, 2000.
[9] S.-H. Suh, E.-S. Lee, and S.-Y. Jung, “Error modelling and measurement for the rotary table of five-axis machine tools,” *Int. J. Adv. Manuf. Technol.*, vol. 14, no. 9, pp. 656–663, 1998.
[10] D. Gu, J. Randa, and D. K. Walker, “A geometric error model for misaligned calibration target in passive microwave remote-sensing systems,” *IEEE Geosci. Remote Sens. Lett.*, vol. 10, no. 6, pp. 1597–1601, Nov. 2013.
[11] B. Zi, H. F. Ding, X. Wu, and A. Kecskeméthy, “Error modeling and sensitivity analysis of a hybrid-driven based cable parallel manipulator,” *Precision Eng.*, vol. 38, no. 1, pp. 197–211, 2014.

[12] K. K. Tan, S. N. Huang, S. Y. Lim, Y. P. Leow, and H. C. Liaw, "Geometrical error modeling and compensation using neural networks," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 36, no. 6, pp. 797–809, Nov. 2006.

[13] W. J. Tian, W. G. Gao, D. Zhang, and T. Huang, "A general approach for error modeling of machine tools," *Int. J. Mach. Tools Manuf.*, vol. 79, pp. 17–23, Apr. 2014.

[14] Y. Lin and Y. Shen, "Modeling of five-axis machine tools metrology models using the matrix summation approach," *Int. J. Adv. Manuf. Technol.*, vol. 21, no. 4, pp. 243–248, 2003.

[15] H. Tang, J.-A. Duan, S. H. Lan, and H. Shui, "A new geometric error modeling approach for multi-axis system based on stream of variation theory," *Int. J. Mach. Tools Manuf.*, vol. 92, pp. 41–51, May 2015.

[16] J. J. Shi and S. Y. Zhou, "Quality control and improvement for multistage systems: A survey," *IIE Trans.*, vol. 41, no. 9, pp. 744–753, 2009.

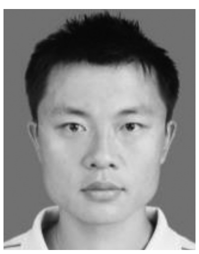
[17] S. Y. Zhou, Y. Chen, and J. J. Shi, "Root cause estimation and statistical testing for quality improvement of multistage manufacturing processes," *IEEE Trans. Autom. Sci. Eng.*, vol. 1, no. 1, pp. 73–83, Jul. 2004.

[18] D. Djurdjanovic and J. Ni, "Stream-of-variation (SoV)-based measurement scheme analysis in multistation machining system," *IEEE Trans. Autom. Sci. Eng.*, vol. 3, no. 4, pp. 407–422, Oct. 2006.

[19] D. Djurdjanovic and J. Ni, "Linear state space modeling of dimensional machining errors," *Trans. NAMRI/SME*, vol. 29, pp. 541–548, 2001.

[20] H. M. Yang, C. T. Chen, R. Ro, and T. C. Liang, "Investigation of the efficient coupling between a highly elliptical Gaussian profile output from a laser diode and a single mode fiber using a hyperbolic-shaped microlens," *Opt. Laser Technol.*, vol. 42, no. 6, pp. 918–926, 2010.

[21] I. Inasaki, K. Kishinami, and S. Sakamoto, *Shape Generation Theory of Machine Tools-Its Basis and Applications*, Yokendo, Tokyo, Japan, 1997, pp. 95–103.



Hao Tang was born in Changsha, China, in 1988. He received the B.S. degree in mechanical engineering from Dong Hua University, Shanghai, China, and the M.S. degree in mechanical engineering from the College of Mechanical and Electrical Engineering, Central South University, Changsha, China, in 2009, where he is currently pursuing the Ph.D. degree.

He was a visiting student with the University of Michigan, Ann Arbor, MI, USA, from 2013 to 2015, and was also involved with S. M. Wu Manufacturing Research Center. Since 2016, he has been with the Hunan University of Science and Technology, Xiangtan, China. His current research interests include error analysis, error modeling and precision transferring in complicated multiaxis motion systems, and applications of optoelectronic packaging systems and laser welding systems.

Ji-An Duan received the Ph.D. degree in mechanical engineering from Xi'an Jiaotong University, Xi'an, China, in 1996.

Since 1996, he has been with the Central South University, Changsha, China, where he is currently a Professor of Mechanical Engineering. His current research interests include precision engineering, photoelectron manufacturing engineering, design and control of electromechanical systems, and dynamic analysis for mechanical systems.

Prof. Duan was named the Cheung Kong Scholar by the Education Ministry of China in 2012. He is also the Director of the State Key Laboratory of High Performance Complex Manufacturing.

Shengqiang Lu was born in Jinhua, Zhejiang, China, in 1989. He received the B.S. and M.S. degrees in mechanical engineering from the College of Mechanical and Electrical Engineering, Central South University, Changsha, China, in 2011 and 2014, respectively, where he is currently pursuing the Ph.D. degree in mechanical engineering.

His current research interests include laser welding and complex structure design.